Abstract

This TeX file serves as a location to collect reference formulas and derivations for Venture SP implementations. Ideally, premises of derivations would be copied in from cited sources verbatim, so we can check the correspondence easily (and don't depend on any transient sources to stick around). Also ideally, a given piece of code should be as direct a translation of the conclusion of a derivation as possible—same parameterization, same variable naming, etc. As of this writing, this document is far from complete.

1 Continuous

1.1 Non-conjugate normal with sufficient statistics

`SuffNormalOutputPSP`

1.1.1 Log density of counts

`http://www.encyclopediaofmath.org/index.php/Sufficient_statistic` says that the likelihood of a vector $\{x_i\}$ of n Gaussian observations with given mean μ and variance σ^2 is

$$p_{\mu,\sigma^2}(\{x_i\}) = (2\pi\sigma^2)^{-n/2} \exp\left(-\frac{n\mu^2}{2\sigma^2} - \frac{1}{2\sigma^2}\sum_{i=1}^{n} x_i^2 + \frac{\mu}{\sigma^2}\sum_{i=1}^{n} x_i\right).$$

Therefore, letting

$$x_{\text{sum}} = \sum_{i=1}^{n} x_i \qquad x_{\text{sumsq}} = \sum_{i=1}^{n} x_i^2$$

we have

$$\log p_{\mu,\sigma^2}(\{x_i\}) = -\frac{n}{2}(\log(2\pi) + 2\log(\sigma)) - \frac{n\mu^2}{2\sigma^2} - \frac{1}{2\sigma^2}x_{\text{sumsq}} + \frac{\mu}{\sigma^2}x_{\text{sum}}.$$

1.1.2 Gradient of log density of counts

Gradient with respect to μ, σ, attached at `MakerSuffNormalOutputPSP`.

$$\frac{d}{d\mu}\log p_{\mu,\sigma^2}(\{x_i\}) \quad = \quad -\frac{n\mu}{\sigma^2} + \frac{x_{\text{sum}}}{\sigma^2}.$$

To differentiate with respect to σ it helps to group terms. Letting

$$x_{\text{sq-dev}} = \sum_{i=1}^{n}(x_i - \mu)^2 = n\mu^2 + x_{\text{sumsq}} - 2\mu x_{\text{sum}},$$

we have

$$\log p_{\mu,\sigma^2}(\{x_i\}) = -\frac{n}{2}(\log(2\pi) + 2\log(\sigma)) - \frac{x_{\text{sq-dev}}}{2\sigma^2}$$

and

$$\frac{d}{d\sigma}\log p_{\mu,\sigma^2}(\{x_i\}) = -\frac{n}{\sigma} + \frac{x_{\text{sq-dev}}}{\sigma^3}$$

This cross-checks with `http://aleph0.clarku.edu/~djoyce/ma218/meeting12.pdf`

1.1.3 Upper bound of log density of counts

Setting the above to zero and solving for μ, σ gives (for $n > 0$)

$$\begin{aligned}
\hat{\mu} &= \frac{x_{\text{sum}}}{n} \\
\hat{\sigma}^2 &= \frac{1}{n}\left(n\hat{\mu}^2 + x_{\text{sumsq}} - 2\hat{\mu}x_{\text{sum}}\right) \\
&= \frac{x_{\text{sum}}^2}{n^2} + \frac{x_{\text{sumsq}}}{n} - 2\frac{x_{\text{sum}}^2}{n^2} \\
&= \frac{x_{\text{sumsq}}}{n} - \frac{x_{\text{sum}}^2}{n^2}
\end{aligned}$$